

137 Expressions for the Fine Structure Constant

Stephen McCormick

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The inverse fine-structure constant α^{-1} is measured to be approximately 137.035999 (measurements disagree in later significant figures). Evaluated to the same number of significant figures, each of these expressions below is equal to the same value of 137.035999. All found by AI with a very short and simple search. Constants used are given on the last page.

$$(1) \quad 90 \sqrt{\delta_{\text{G}}} \beta \sigma$$

$$(2) \quad \frac{14}{\Omega C_{\text{CE}} K_{\text{LR}}}$$

$$(3) \quad \frac{94 \sqrt{A^5} \zeta(5)}{\rho}$$

$$(4) \quad \frac{54}{C_{\text{A}}} - 7 \chi - M$$

$$(5) \quad 64 \theta^{-\Omega} P_{\text{U}}^{\ln 3}$$

$$(6) \quad \frac{5 \sqrt{\sigma^7}}{\sqrt{(\ln 2)^3} C_{\text{A}}}$$

$$(7) \quad \frac{83}{\delta_{\text{G}}} - \varpi + \frac{1}{W}$$

$$(8) \quad 4 G^{C_{\text{A}}} B^{8\beta}$$

$$(9) \quad \frac{4 \lambda P_{\text{U}}^6}{\sqrt{\pi^3}}$$

$$(10) \quad \frac{68}{(\ln 2)^2} + \alpha_{\text{F}} - 7$$

$$(11) \quad \delta^{\sigma/2} \gamma^{-5\rho}$$

$$(12) \quad \frac{95 \varphi^4}{\sqrt{2} \psi}$$

$$(13) \quad \sqrt{\frac{7745}{\tau}} + \chi$$

$$(14) \quad 2 \lambda_{\text{L}}^{\lambda} K_0^{5/\zeta(5)}$$

$$(15) \quad \frac{98}{C_2} - \tau - 11$$

$$(16) \quad 83 \lambda_{\text{L}}^{-M} K_{\text{S}}^P$$

$$(17) \quad 852 \mu \ell \sqrt{\chi}$$

$$(18) \quad 97 E_{\text{B}} + \beta - 20$$

$$(19) \quad \frac{\sqrt{\delta_{\text{S}}^9}}{8 (\ln 2)^9 \lambda}$$

$$(20) \quad 3 C_{10}^{-T} \gamma^{C_{\text{CE}}/5}$$

$$(21) \quad 9 \rho + \lambda_{\text{L}} + 88 \sqrt{2}$$

$$(22) \quad \frac{449}{4 \sqrt{\varphi} \sqrt{P}}$$

$$(23) \quad \frac{74}{\Omega} + \frac{8}{\chi} - \rho$$

$$(24) \quad 9 \Psi^{\rho} B^{7/\beta}$$

$$(25) \quad 31 \delta_{\text{S}}^{-\ell} \psi^{\theta}$$

$$(26) \quad 64 W + E_{\text{B}} + \frac{2}{\mu}$$

$$(27) \quad \frac{877 \sqrt{C_{\text{CE}}} T}{5 \sqrt{\theta}}$$

$$(28) \quad 84 \varphi + \frac{4}{\Gamma(\frac{1}{4})} + M^3$$

$$(29) \quad 93 P_{\text{U}}^{\chi} T^{-2C_{\text{A}}}$$

$$\begin{aligned}
(30) \quad & \frac{32}{C_{\text{CE}}} + A - 8 C_{\text{CE}}^5 & (51) \quad & \frac{32 \delta^2 \sqrt{\varpi}}{9 G} \\
(31) \quad & \frac{148 \sqrt{G^5} K_{\text{LR}}}{\lambda_{\text{L}}} & (52) \quad & 565 \pi^{-\ln 3} C_{\text{CE}}^{\ell} \\
(32) \quad & 96 T - 5 \sigma^4 - \sqrt{2} & (53) \quad & 772 \tau^2 + A^7 \\
(33) \quad & 71 K_0^{C_{\text{CE}}} \Psi^9 C_{10} & (54) \quad & 81 E_{\text{B}} K_{\text{S}}^{5/92} \\
(34) \quad & \frac{52 \delta_{\text{G}} P_{\text{U}}^6}{\sqrt{e^7}} & (55) \quad & 32 \varphi^{-\rho} e^{8 M} \\
(35) \quad & \frac{156}{\ln 3} - 8 \tau - \sigma & (56) \quad & 54 P_{\text{U}} + 16 \Omega + 4 \\
(36) \quad & 44 \zeta(5)^{-\lambda} C_{\text{N}}^{3 \eth} & (57) \quad & \frac{\sqrt{B^{63}}}{41 C_{\text{N}}^6} \\
(37) \quad & \frac{\sqrt{K_0^9}}{\sqrt{\tau^3} E_{\text{B}}^{9/5}} & (58) \quad & \sqrt{\frac{7}{C_{\text{A}}^8} + \delta^4 - M^4} \\
(38) \quad & 36 \lambda^5 + \delta - \Psi^3 & (59) \quad & 53 E_{\text{B}}^{F/2} \beta^{\sigma} \\
(39) \quad & \frac{5 \pi^{C_{\text{N}}} \lambda_{\text{L}}^{-5 E_{\text{B}}}}{7} & (60) \quad & \frac{\sqrt{\zeta(3)^{49}} \lambda}{G^{5/3}} \\
(40) \quad & \frac{54 \sqrt{L_{\text{K}}}}{\lambda \delta_{\text{G}}^{7/6}} & (61) \quad & \frac{348}{\alpha_{\text{F}}} - T - \eth^6 \\
(41) \quad & 194 \ln 2 + 3 \lambda_{\text{L}} + \gamma & (62) \quad & \frac{\sqrt{\lambda_{\text{D}}}}{3 \Gamma(\frac{1}{4})^6 K_{\text{LR}}^{52}} \\
(42) \quad & 97 \eth^{L_{\text{K}}} \xi^{-2/C_2} & (63) \quad & 188 E_{\text{B}}^{-\Omega} \zeta(5)^{-\lambda} \\
(43) \quad & 82 \sigma + \frac{6}{\rho^5} - \ln 2 & (64) \quad & 96 B - 3 \chi + \frac{1}{\psi} \\
(44) \quad & \frac{\sqrt{\kappa}}{4 \sqrt{\beta} \gamma^{68/5}} & (65) \quad & \frac{\sigma g^{25/7}}{\sqrt{C_{\text{CE}}^7}} \\
(45) \quad & \frac{356}{K_{\text{S}}} - \Gamma(\frac{1}{4}) + \frac{8}{e} & (66) \quad & 95 \varphi - \frac{5}{\xi^3} + W \\
(46) \quad & \frac{97 \delta_{\text{S}}^{8/9}}{\beta \sqrt{C_{\text{N}}}} & (67) \quad & 62 \beta^{-B} \Psi^{4/C_{\text{P}}} \\
(47) \quad & 203 \lambda_{\text{L}} + \pi - \frac{3}{\delta} & (68) \quad & \frac{\zeta(3)^{59} K_{\text{S}}}{2 F^6} \\
(48) \quad & 29 \lambda_{\text{D}}^{-P} L_{\text{K}}^{3/\varpi} & (69) \quad & 42 L_{\text{K}} - \eth + 8 C_2^9 \\
(49) \quad & 4 \alpha_{\text{F}} C_{\text{P}}^6 W^{3/7} & (70) \quad & 193 K_0^{\beta} F^{-C_{\text{P}}} \\
(50) \quad & \frac{544}{\beta^8} - g - \Omega & (71) \quad & 87 E_{\text{B}} - \frac{4}{\mu} + \delta_{\text{G}}^9 \\
(51) \quad & & (72) \quad & \frac{\sqrt{\theta^{69}} L_{\text{K}}}{86 F}
\end{aligned}$$

$$(73) \quad 366 \gamma^{-C_A} W^{-E_B}$$

$$(74) \quad \sqrt{\frac{25}{\ell^3} - \frac{8}{F} - \xi}$$

$$(75) \quad \frac{74 E_B}{\rho^{4/3} \delta_G}$$

$$(76) \quad \frac{63}{K_{LR}^3} - K_S - 2 K_{LR}$$

$$(77) \quad \frac{124 \zeta(3)^2}{5 M}$$

$$(78) \quad \frac{9}{\xi^6} + \frac{8}{A} + \frac{4}{\chi}$$

$$(79) \quad 35 \mu^{K_S} W^{1/T}$$

$$(80) \quad \frac{\sqrt{F}}{57 \varphi \sqrt{C_2^{43}}}$$

$$(81) \quad 244 \delta_G - 5 W + 2$$

$$(82) \quad 32 G^{-\alpha_F} K_0^{5/4}$$

$$(83) \quad \frac{53}{C_2} + 6 W^3 + \varphi$$

$$(84) \quad \frac{60 \sqrt{\delta_G}}{7 \theta \lambda_D^7}$$

$$(85) \quad 706 \delta^{-P} \mu^{-K_0}$$

$$(86) \quad \frac{15 \sqrt{\Gamma(\frac{1}{4})^3} \zeta(5)^2}{\sqrt{C_2}}$$

$$(87) \quad \frac{66}{P} - E_B^7 + 3 T$$

$$(88) \quad 83 \pi^{-\mu} \tau^{-8/L_K}$$

$$(89) \quad \frac{964 A}{\delta_S^5 \ell}$$

$$(90) \quad T^8 + \frac{4}{C_2} - \frac{2}{K_0^9}$$

$$(91) \quad \frac{191 \theta L_K}{9 \lambda_L}$$

$$(92) \quad 670 \Gamma(\frac{1}{4})^{-\beta} C_N^{-\ell}$$

$$(93) \quad \frac{87}{G} + 2 \varpi^3 + 6$$

$$(94) \quad \frac{72 \theta}{7 \sqrt{\lambda_L^9} \lambda_D}$$

$$(95) \quad 22 \delta + 8 e + 6 W$$

$$(96) \quad \frac{5 \sqrt{\varphi^{21}} \delta_G^3}{\sqrt{\Psi}}$$

$$(97) \quad 51 M^{-\xi} \sigma^{2 C_{10}}$$

$$(98) \quad \frac{84 \delta_S \theta}{L_K^{5/9}}$$

$$(99) \quad 70 C_N + 6 \psi - 6 P$$

$$(100) \quad \frac{190 \sqrt{\sigma} \sqrt{P_U} K_S}{7}$$

$$(101) \quad 20 T^\psi \mu^{-3 \ell}$$

$$(102) \quad \sqrt{7 K_0^8 - 56 e - K_0}$$

$$(103) \quad \frac{P_U^{53/6}}{\theta^9 \chi}$$

$$(104) \quad 81 \mu + \frac{4}{C_{10}} - 5 K_S$$

$$(105) \quad \frac{563 \pi^{-K_0}}{C_2^4}$$

$$(106) \quad \frac{455}{L_K} - \frac{2}{\varphi} - \lambda_D$$

$$(107) \quad \sqrt{\frac{6}{M^6} + \frac{6}{\ln 2} + (\sqrt{3})^3}$$

$$(108) \quad \frac{508}{\psi} - \frac{7}{K_{LR}} - 5$$

$$(109) \quad \frac{90 P_U}{\sqrt{\zeta(3)} \Psi^{5/6}}$$

$$(110) \quad \frac{58 \alpha_F^\varpi \delta_S^\vartheta}{9}$$

$$(111) \quad 59 P_U + \theta^2 - \xi^5$$

$$(112) \quad 611 \varphi^{-e} \zeta(3)^{-\chi}$$

$$(113) \quad \frac{\psi K_S^{59/8}}{27}$$

$$(114) \quad 8 F^{1/\lambda_L} C_N^{2 \zeta(3)}$$

$$(115) \quad \frac{195}{A} - \alpha_{\text{F}}^3 + C_2$$

$$(116) \quad \frac{48 \pi \sqrt{\delta}}{\varphi^{9/5}}$$

$$(117) \quad 51 K_{\text{S}} + 8 \xi + C_2^7$$

$$(118) \quad 553 \Gamma(\tfrac{1}{4})^\kappa \lambda_{\text{D}}^{L_{\text{K}}}$$

$$(119) \quad \frac{\xi (\ln 3)^3}{5 g^{37}}$$

$$(120) \quad 3 C_{\text{N}}^{-1/\Psi} C_2^{-3\psi}$$

$$(121) \quad \frac{727}{P_{\text{U}}^2} - \zeta(5) + \kappa$$

$$(122) \quad \frac{9 A^8 W}{\chi^{3/7}}$$

$$(123) \quad 94 C_{\text{P}} - \zeta(3) + 3 (\ln 2)^6$$

$$(124) \quad \frac{611 \sqrt{\gamma} A \sqrt{\rho}}{5}$$

$$(125) \quad \frac{90 B^{K_{\text{S}}}}{\delta_{\text{S}}^{5/8}}$$

$$(126) \quad \Gamma(\tfrac{1}{4})^4 - \frac{40}{\delta} + 8 P_{\text{U}}$$

$$(127) \quad \frac{567 \sqrt{\sigma}}{2 \sqrt{\lambda_{\text{L}}} L_{\text{K}}}$$

$$(128) \quad 274 K_{\text{LR}} - 79 G$$

$$(129) \quad \frac{\xi^{62/3}}{C_{10}^7 T}$$

$$(130) \quad \frac{19}{\ell} - \frac{93}{\alpha_{\text{F}}} + C_{\text{P}}$$

$$(131) \quad 89 \lambda^{-\lambda_{\text{D}}} \theta^{6/K_0}$$

$$(132) \quad \frac{9}{\kappa} + 31 \chi + \frac{3}{\ell}$$

$$(133) \quad 275 C_{\text{CE}}^{K_{\text{LR}}} C_{\text{A}}^{-P}$$

$$(134) \quad \frac{961 \ln 3}{7 \sqrt{g} \sqrt{\mu}}$$

$$(135) \quad \frac{923 \sqrt{\ln 2}}{9 \Omega \ln 3}$$

$$(136) \quad 4 \delta_{\text{S}}^4 + 2 (\ln 2)^7 + 1$$

$$(137) \quad \frac{79}{\gamma} + \frac{4}{\Psi^3} - \ln 3$$

Notation

Symbol	Name	Value
π	Archimedes' constant	3.141592653590
e	Euler's number	2.718281828459
γ	Euler–Mascheroni constant	0.5772156649015
φ	golden ratio	1.618033988750
$\ln 2$	natural logarithm of 2	0.6931471805599
$\ln 3$	natural logarithm of 3	1.098612288668
$\zeta(3)$	Apéry's constant	1.202056903160
$\zeta(5)$	Riemann zeta at 5	1.036927755143
G	Catalan's constant	0.9159655941772
K_0	Khinchin's constant	2.685452001065
A	Glaisher–Kinkelin constant	1.282427129101
δ	first Feigenbaum constant	4.669201609103
α_F	second Feigenbaum constant	2.502907875096
ρ	plastic number ($\rho^3 = \rho + 1$)	1.324717957245
δ_S	silver ratio ($1 + \sqrt{2}$)	2.414213562373
Ω	omega constant ($\Omega e^\Omega = 1$)	0.5671432904098
$\eth$	Dottie number ($\cos \eth = \eth$)	0.7390851332152
g	Gauss's constant	0.8346268416741
ϖ	lemniscate constant	2.622057554292
M	Meissel–Mertens constant	0.2614972128476
C_2	twin-prime constant	0.6601618158469
λ	Conway's look-and-say constant	1.303577269034
λ_L	Laplace limit	0.6627434193492
δ_G	Euler–Gompertz constant	0.5963473623232
$\Gamma(\frac{1}{4})$	Gamma function at $\frac{1}{4}$	3.625609908222
C_{10}	Champernowne constant	0.1234567891011
C_{CE}	Copeland–Erdős constant	0.2357111317192
P	prime constant ($\sum_p 2^{-p}$)	0.4146825098511
K_{LR}	Landau–Ramanujan constant	0.7642236535892
E_B	Erdős–Borwein constant	1.606695152415
θ	Mills' constant (assuming RH)	1.306377883863
β	Lévy's constant ($\pi^2/(12 \ln 2)$)	1.186569110416
L_K	Khinchin–Lévy constant (e^β)	3.275822918722
C_P	Porter's constant	1.467078079434
σ	Somos' quadratic recurrence constant	1.661687949634
C_N	Niven's constant	1.705211140105
ψ	reciprocal Fibonacci constant ($\sum 1/F_n$)	3.359885666243
P_U	universal parabolic constant	2.295587149393
μ	Ramanujan–Soldner constant ($\text{li } \mu = 0$)	1.451369234883
τ	Prouhet–Thue–Morse constant	0.4124540336401
T	tribonacci constant ($T^3 = T^2 + T + 1$)	1.839286755214
Ψ	supergolden ratio ($\Psi^3 = \Psi^2 + 1$)	1.465571231877
W	Wallis's constant ($W^3 = 2W + 5$)	2.094551481542
K_S	Sierpiński's constant	2.584981759579
ℓ	Liouville's constant ($\sum 10^{-n!}$)	0.1100010000000
B	Backhouse's constant	1.456074948583
C_A	Artin's constant	0.3739558136192
λ_D	Golomb–Dickman constant	0.6243299885436
ξ	Cahen's constant	0.6434105462883
κ	Kepler–Bouwkamp constant ($\prod \cos(\pi/n)$)	0.1149420448533
χ	Gieseking's constant	1.014941606410
F	Fransén–Robinson constant ($\int_0^\infty dx/\Gamma(x)$)	2.807770242029